Amplified spontaneous emission. II. The connection with laser theory

1971 J. Phys. A: Gen. Phys. 4 377

(http://iopscience.iop.org/0022-3689/4/3/015)

J. Phys. A: Gen. Phys., 1971, Vol. 4. Printed in Great Britain

Amplified spontaneous emission
II. The connection with laser theory

L. ALLEN and G. I. PETERS

School of Mathematical and Physical Sciences, University of Sussex, Falmer, Brighton, England

MS. received 21*st November* 1970, *in revised form* 4*th January* 1971

Abstract. A simple relationship is derived between L_c—the critical length for amplified spontaneous emission, an expression for which has been previously derived by the authors—and L_T, the threshold length of gas that must be excited for laser action to occur in a cavity of loss δ_l. The relationship is experimentally verified and, in passing, the semiclassical laser theory of Stenholm and Lamb is re-verified in a new way.

1. Introduction

Recently (Allen and Peters 1970, Peters and Allen 1971 to be referred to as I) we have derived the threshold condition for the onset of amplified spontaneous emission (ASE), which is the highly directional radiation emitted by a high-gain system in the absence of a laser cavity. In this work we derive a relationship between the critical length L_c for ASE to occur, and L_T, the threshold length of gas that must be excited for the same type of atoms to give laser oscillation in a conventional laser cavity. The value for L_c is that derived from our earlier work, and the value for L_T is obtained from the semiclassical theory of Stenholm and Lamb (1969) which was derived to be applicable to lasers of any level of intensity.

The relationship is experimentally verified by use of the 3·39 μm He–Ne system and, in passing, the Stenholm and Lamb theory is verified independently by showing that it predicts the correct relationship between output intensity and inversion density. This is an alternative verification of the theory already shown to be valid in an earlier paper (Allen 1969) for all regions except very close to threshold where the failure to account adequately for spontaneous emission leads to a slight discrepancy between theory and experiment.

This work also allows the direct comparison of the output intensities achievable from the laser and by ASE for a given inversion density. For any given inversion density, laser action will invariably occur with a shorter length of excited gas than that required to give ASE. However, the small transmission coefficients of the required mirrors allow the output intensity, though not of course the degree of coherence, to be greater from ASE than from laser action—even for discharge lengths not greatly in excess of L_c.

2. Theory

Combining equations (12) and (98) from the paper of Stenholm and Lamb, and assuming a steady state such that $\dot{E} = 0$, yields

$$(1+I)^{1/2} = \frac{Q\pi^{1/2}\mathscr{P}^2\bar{N}}{\hbar K u \epsilon_0}$$

for a laser oscillation sufficiently close to the centre of the gain curve that

$|\omega-\nu| \ll \gamma_{ab}$, where I is the laser intensity within the cavity, $\mathscr{P}$ the dipole moment matrix element, $\bar{N}$ the inversion density, Q the quality factor of the cavity and Ku is a factor which arises from the Maxwellian distribution of atom velocities and is equal to 0·6 times the Doppler width $\Delta\nu_D$ of the atomic transition involved. But

$$Q \simeq \frac{1}{\delta_l}\frac{2\pi l_c}{\lambda}$$

where δ_l is the fractional loss per pass in the laser resonator and l_c is the cavity length. Therefore

$$(1+I)^{1/2} = \frac{1}{\delta_l}\frac{2\pi^{3/2}}{\lambda}\frac{\mathscr{P}^2}{\hbar K u \epsilon_0} l_c \bar{N}.$$

The population inversion $\bar{N}$ is given by

$$\bar{N} = \frac{1}{l_c}\int_0^{l_c} N(z, t)\,dz$$

so it is trivially shown that $\bar{N} l_c = \bar{n} L$ where $\bar{n}$ is the inversion density in the excited part of the cavity rather than that integrated over the whole cavity, and L is the length of discharge.

Thus

$$(1+I)^{1/2} = \frac{1}{\delta_l}\frac{2\pi^{3/2}\mathscr{P}^2}{\lambda\hbar K u \epsilon_0} L\bar{n}. \tag{1}$$

At threshold $I = 0$ and $L = L_T$, where L_T is the length of active discharge at which laser action begins, and so, substituting for Ku and $\mathscr{P}^2$ in terms of the Doppler width and atomic lifetime of the transition,

$$\bar{n} L_T = 3\cdot 2\pi^{3/2}\frac{\Delta\nu_D \tau_2 \delta_l}{\lambda^2 \phi}.$$

However, we have shown previously (see I) that for the same inversion density of atoms $\bar{n}$ we can write the critical length L_c for the onset of ASE as

$$\bar{n} L_c = 8\pi \frac{\Delta\nu_D \tau_2}{\lambda^2 \phi}$$

where $\Delta\nu_D$ is the Doppler width, τ_2 is the lifetime of the upper level and ϕ the branching ratio of the transition.

Thus,

$$\frac{L_T}{L_c} = 0\cdot 71\,\delta_l. \tag{2}$$

From (1) it is clear that

$$(1+I)^{1/2} = k_1 L \bar{n}$$

and for $I = 0$ at $L = L_T$ this implies that

$$k_1 \bar{n} = \frac{1}{L_T}$$

therefore

$$(1+I)^{1/2} = \frac{L}{L_T}.$$

However, in practice it is difficult to measure the laser intensity directly and I must be replaced by a photocurrent i times a scaling constant k. The scaling constant automatically takes care of the transmission coefficient of the output mirror as well as the parameters of the detection system.

Thus

$$i = \frac{L^2}{kL_T^2} - \frac{1}{k}. \tag{3}$$

If equation (3) can be verified experimentally then the theory due to Stenholm and Lamb is verified; if equation (2) is verified then an independent verification of our ASE threshold condition will be achieved in addition to the direct verification described previously in I.

3. Comparison of theory with experiment

The 3·39 μm transition in He–Ne was employed as it was known that it would operate easily both in a laser and as a source of ASE. A 125 cm long He–Ne discharge tube was placed between two high-reflectivity concave mirrors, $R = 0{\cdot}98$, and of radius of curvature $r = 3$ m. The gas was rf excited and it was possible to excite a variable length of tube by removing or adding rf electrodes. The inversion density in the tube was controlled by monitoring the spontaneous emission from the upper level of the transition as discussed in I. As electrodes were added or removed, careful adjustment of the rf input was necessary to ensure that the inversion density of the gas remained constant.

The output intensity was measured as a function of length of tube excited, for three inversion densities. For each of these inversions the mirror at the detector end was removed and the amplified spontaneous emission observed as a function of length of excited tube (see Allen and Peters 1971 paper III in this series). In practice, rather than fix the inversion and determine L_T and L_c for this inversion, L_T was fixed and the corresponding value of L_c determined. This was necessitated by the fact that L_T is small, approximately 5–10 cm for the inversions obtainable, and length variations were in minimum steps of 2·5 cm so that, if the inversion was first of all fixed, errors in determining L_T could be very large. After fixing L_T the rf power was adjusted to bring the laser to threshold and then this inversion was maintained for the intensity against length determinations for both laser and ASE.

Figure 1 shows, for three values of inversion density, the way in which the output intensities of the laser and of ASE vary with the length of the excited gas. The laser threshold length is L_T, and the threshold length L_c for ASE is twice the cut-off discharge length shown in figure 1 as one mirror is still in position. It may be noticed that, at an effective length of only approximately 1·5 L_c, the output intensity is greater from the ASE arrangement than from the laser. This is simply due to the 1% transmission coefficient of the laser mirror. It must not be thought that the outputs have similar temporal coherence properties; the linewidth in the laser will be appreciably narrower than that of the ASE.

Figure 2 shows a plot of laser output intensity against L^2. The straight line obtained for each of the three inversion densities used confirms equation (3) and

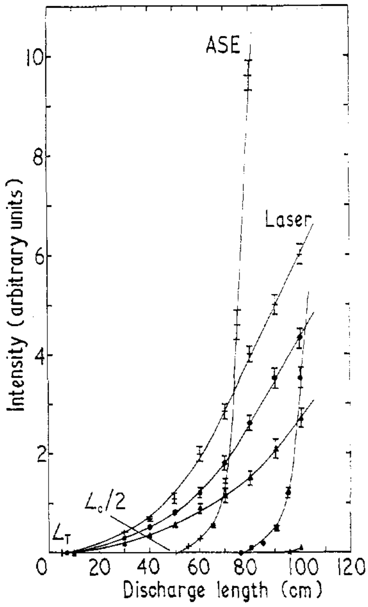

Figure 1. Plots of laser and ASE itensities against discharge length, for three inversion densities. The curve drawn with crosses has the highest inversion density and that with triangles the lowest.

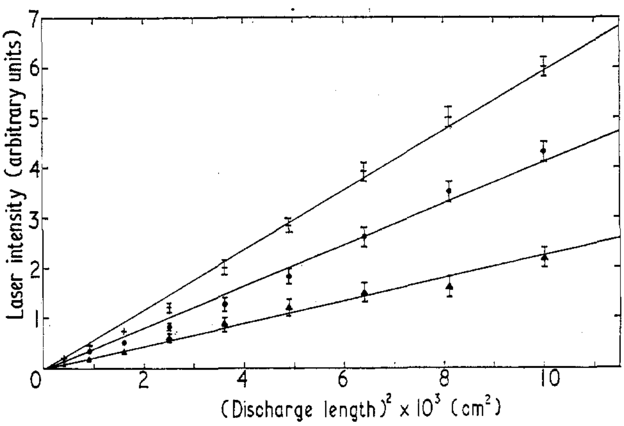

Figure 2. Plots of laser output against the square of the discharge length, for three inversion densities.

verifies the theory of Stenholm and Lamb. Equation (2) predicts that L_T should be linear with L_c. Figure 3 is in excellent agreement with this prediction, and the slope of the line is found to be 0·005. This equals 0·71 δ_l, and so δ_l is 0·07. This may be slightly too large in practice. The mirror losses amount to approximately 0·04 and this implies that the individual window losses are approximately 0·015 which seems slightly high. However, the exact value of the numerical constant is somewhat in doubt; presumably, if the lineshape of the Doppler-broadened line could be taken into account in the derivation of L_c, a small numerical change, although of order unity, would take place.

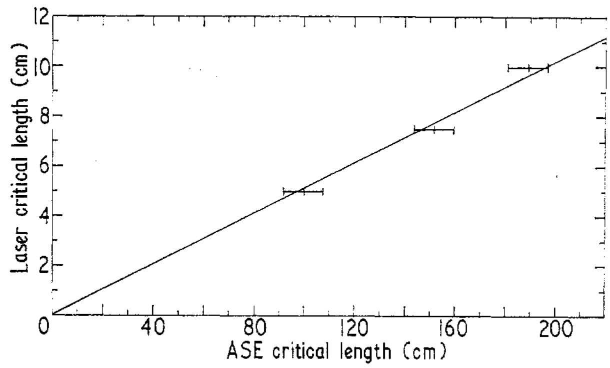

Figure 3. Plot of L_T against L_c.

It should be noted that, to give the comparison shown in figure 1, one mirror is always in place. Thus, the cut-off length for the ASE is designated $L_c/2$. It should be realized, of course, that the effect of the mirror is only to double the length to within an accuracy of perhaps 3–5%. This effect is discussed more fully in paper III.

4. Conclusions

The experimental results from the 3·39 μm He–Ne transition satisfy the theoretical predictions extremely well. In previous verifications of laser theory (Sayers *et al.* 1969, Allen 1969) considerable attention has been paid to the fact that near threshold it is important to take into account the quantum effects due to spontaneous emission. Consequently, one should ordinarily view the value of L_T derivable from the Stenholm and Lamb theory with some caution. Nevertheless, although a few low-intensity points are the least satisfactory of those plotted in figure 2, there seems to be no reason to plot anything other than the best straight line through all available points. Indeed, given the errors implicit in intensity measurement at 3·39 μm, the straight lines are remarkably good.

Why can this ironical worry about spontaneous effects on the laser threshold, and its effect on a comparison with ASE, be ignored? The explanation appears to be that the length of discharge at which L_T occurs is far below the lengths normally occurring in a laser when $L \sim L_c$, and thus, unlike the previous verification (Allen 1969), spontaneous emission is genuinely a very small effect.

The simple connection between L_T, L_c and δ_l may be recognized physically as representing the way in which L_T is effectively enlarged by multiple reflection from the mirrors to the level L_c at which one mode becomes excited. It is nonetheless pleasing to derive the relationship so trivially and to see it confirmed so well.

Acknowledgments

One of us (G.I.P.) wishes to acknowledge the Science Research Council for providing a research studentship.

References

Allen, L., 1969, *J. Phys.* A: *Gen. Phys.*, **2**, 433–5.
Allen, L., and Peters, G. I., 1970, *Phys. Lett.*, **31A**, 95–6.
—— 1971, to be published in *J. Phys.* A: *Gen. Phys.*
Peters, G. I., and Allen, L., 1971, *J. Phys.* A: *Gen. Phys.*, **4**, 238–43.
Sayers, M. D., Allen, L., and Jones, D. G. C., 1969, *J. Phys.* A: *Gen. Phys.*, **2**, 102–5.
Stenholm, S., and Lamb, W. E., Jr., 1969, *Phys. Rev.*, **181**, 618–35.